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Constraints on SUSY lepton flavour violation by rare processes

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Abstract: We study the constraints on flavour violating terms in low energy SUSY coming from several processes as $l_i \to l_j\gamma$, $l_i \to l_j l_j l_j$ and $\mu \to e$ in Nuclei. We show that a combined analysis of these processes allows us to extract additional information with respect to an individual analysis of all the processes. In particular, it makes possible to put bounds on sectors previously unconstrained by $l_i \to l_j\gamma$. We perform the analysis both in the mass eigenstate and in the mass insertion approximations clarifying the limit of applicability of these approximations.

Keywords: Supersymmetry Phenomenology.

Contents

1. Introduction

Neutrino oscillation experiments have established the existence of lepton family number violation processes. So, as a natural consequence of neutrino oscillations, one would expect flavour mixing in the charged lepton sector. This mixing can be manifested in rare decay processes such as $\mu \to e\gamma$, $\tau \to \mu\gamma$, etc. However, if only the lepton yukawa couplings carry this information on flavour mixing, as in the Standard Model with massive neutrinos, the expected rates of these processes are extremely tiny being proportional to the ratio of masses of neutrinos over the masses of the W bosons [1]. These values are very far from the present and upcoming experimental upper bounds that we can read in table 1.

In a supersymmetric (SUSY) framework the situation is completely different [2]. For instance, the supersymmetric extension of the see-saw model [3] provides new direct sources of flavour violation, namely the possible presence of off-diagonal soft terms in the slepton mass matrices and in the trilinear couplings [4]. In practice, flavour violation would originate from any misalignment between fermion and sfermion mass eigenstates.

	Process	Present bounds	Expected future bounds
(1)	$\mathrm{BR}(\mu \to e, \gamma)$	1.1×10^{-11}	$\mathcal{O}(10^{-13} - 10^{-14})$
(2)	$\mathrm{BR}(\mu \to e, e, e\ )$	1.1×10^{-12}	$\mathcal{O}(10^{-13} - 10^{-14})$
(3)	$\mathrm{BR}(\mu \to e$ in Nuclei)	1.1×10^{-12}	$\mathcal{O}(10^{-13} - 10^{-14})$
(4)	$\mathrm{BR}(\tau \to e, \gamma)$	3.1×10^{-7}	$\mathcal{O}(10^{-8})$
(5)	$\mathrm{BR}(\tau \to e, e, e)$	2.7×10^{-7}	$\mathcal{O}(10^{-8})$
(7)	$\mathrm{BR}(\tau \to \mu, \gamma)$	6.8×10^{-8}	$\mathcal{O}(10^{-8})$
(8)	$\mathrm{BR}(\tau \to \mu, \mu, \mu)$	2×10^{-7}	$\mathcal{O}(10^{-8})$

Table 1: Present and Upcoming experimental limits on various leptonic processes [5 – 10].

One of the major problems of low energy supersymmetry is to understand why all LFV processes are so suppressed. This suppression imposes very severe constraints on the pattern of the sfermion mass matrices which must be either very close to the unit matrix in the flavour space (flavour universality) or almost proportional to the corresponding fermion mass matrices (alignment). Both universality and alignment can be either present as a kind of "initial" conditions or as a result from some dynamics of the theory.

Given a specific SUSY model, it is possible, in that context, to make a full computation of all the FCNC (and possibly also CP violating) phenomena. This is the case, for instance, of the constrained minimal supersymmetric standard model (CMSSM) where these detailed computations led to the result of utmost importance that this model succeeds to pass unscathed all the severe FCNC and CP tests. However, given the variety of options that exists in extending the MSSM (for instance embedding it in some more fundamental theory at larger scale), it is important to have a way to extract from the whole host of FCNC and CP phenomena a set of upper limits on quantities which can be readily computed in any chosen SUSY frame. Namely, one needs some kind of model-independent parameterization of the FCNC and CP quantities in SUSY to have an accessible immediate test of variants of the MSSM.

The best parameterization of this kind that has been proposed is in the framework of the so-called mass insertion approximation (MI) [11]. One chooses a basis for the fermion and sfermion states where all the couplings of these particles with neutralinos or with gluinos are flavour diagonal, while the flavour changing is exhibited by the non-diagonality of the sfermion propagators. Denoting with Δ_{ij} the off-diagonal terms (in flavour space) in the sfermion mass matrices, the sfermion propagators can be expanded as a series in terms of $\delta_{ij} = \Delta_{ij}/\tilde{m}^2$ where $\tilde{m}$ is an average sfermion mass squared. As long as δ_{ij} is much smaller than one, we can just take the first term of this expansion and then the experimental information concerning FCNC and CP violating phenomena translates into upper bounds on these δ_{ij}.

Obviously, the above mass insertion method is advantageous because one does not need the full diagonalization of the sfermion mass matrices to perform a test of the considered susy model in the FCNC sector.

Pioneering studies [12, 13] considered only the photinos and gluinos mediated FCNC contributions because a complete inclusion of the neutralino and chargino sector contributions would require a complete specification of the model. On the contrary, the spirit of their study was to provide a model-independent way to make a first check of the FCNC impact on classes of SUSY theories.

Subsequently, general studies in the leptonic sector [14] included the chargino and neutralino contributions using a generalization of the MI, that we call GMI [15], which consists in an approximation where the gaugino-higgsino mixings are also treated as insertions in the propagators of the charginos and neutralinos inside the loop. With this additional insertion approximation, it is not necessary to fix a particular scenario to analyze the dependence on the many mass parameters. However, in the lepton sector, interferences among amplitudes are generically present in the models and they would affect the limits on the δ_{ij}. This happens, for instance, for δ^{RR} due to a destructive interference between the bino and bino-higgsino amplitudes. Moreover $\tau \to \mu\gamma$ and $\tau \to e\gamma$ experimental bounds are not so stringent as in $\mu \to e\gamma$ so that we can have not so tiny insertions independently from cancellations.

In this context, the target of this work is to study all the possible constraints on flavour violating terms in low energy SUSY coming from several processes as $l_i \to l_j\gamma$, $l_i \to l_j l_j l_j$ and $\mu \to e$ in Nuclei and, subsequently, to clarify which is the limit of applicability of the MI and of its generalization.

The approach is to fix a specific model of known spectrum (CMSSM) and to compare, systematically, the predictions of the full computation [16] with respect to the approximate, MI and GMI, computations [18, 14].

In particular, we focused on the RR sector where strong cancellations make the sector unconstrained. We show that these cancellations prevent us from getting a bound in the RR sector both at the present and also in the future when the experimental sensitivity on $Br(l_i \to l_j\gamma)$ will be increased. So, being our aim to find constraints in the RR sector, we examined other LFV processes as $l_i \to l_j l_j l_j$ and $\mu \to e$ in Nuclei.

Finally we took into consideration the bounds on the various double mass insertions $\delta_{23}^{AB}\delta_{31}^{CD}$, with $A, B, C, D = L, R$. These limits are important in order to get the largest amount of information on SUSY flavour symmetry breaking.

2. Lepton flavour violation in supersymmetry

The study of lepton flavour violation in SUSY scenarios is one of the most promising subjects in low energy supersymmetric phenomenology, in that it shows, if observed, a clear signal of physics beyond the Standard Model [19].

The source of LFV is the soft supersymmetry breaking lagrangian which, in general, contains a too large number of FV couplings, so that the predicted branching ratios for LFV processes usually exceed phenomenological constraints [21]: this is a typical example of the SUSY flavour-problem.

The usual way to solve this problem is to consider lagrangians which result from models that break SUSY in a flavour blind manner, as in mSUGRA or Anomaly mediated supersymmetry breaking (AMSB), etc [22].

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Even in this case, in general, flavour is violated in the lagrangian at the weak scale. For instance, LFV can be induced by the existence of new particles at high scale with flavour violating couplings to the SM leptons (as right handed neutrinos in a see-saw model [12]) or the presence of new Yukawa interactions, as in superGUTs theories [23, 24]. In this last case the flavour violation is communicated to low energy fields through renormalization effects.

On the other hand, in models based on supergravity or superstring theories, nonuniversal soft terms are generically present in the high scale effective lagrangian [25].

In models with flavour symmetry imposed by a Froggatt-Nielsen mechanism, flavour violating corrections to the soft potential could be large [26 − 28].

Irrespective to the source of these FV entries, our approach is to assume LFV at low energy and to try to bound the FV terms present in slepton mass matrices.

The processes we are going to study are $l_i \to l_j\gamma$, $l_i \to l_j l_j l_j$ and $\mu \to e$ in Nuclei that are mediated, at one loop level, by neutralinos, charginos and sleptons. The relevant interaction lagrangian for the considered processes is:

$$\mathcal{L} = \bar{l}_i \left(C^R_{iAX} P_R + C^L_{iAX} P_L\right) \tilde{\chi}^-_A \, \tilde{\nu}_X + \bar{l}_i \left(N^R_{iAX} P_R + N^L_{iAX} P_L\right) \tilde{\chi}^0_A \tilde{l}_X \,. \tag{2.1}$$

We can always work in the basis where the charged lepton masses and the gauge couplings are flavour diagonal. In this basis, in general, the slepton mass matrix is not diagonal and its off-diagonal entries induce the LFV.

Our aim is to use the couplings $C^{R,L}_{iAX}$ and $N^{R,L}_{iAX}$, which are functions of the FV entries, to constrain the structure of the slepton mass matrix.

3. Mass insertion approximation

In the spirit of the mass insertion approximation, we treat the off-diagonal elements of the slepton mass matrix as insertions. Our convention for the slepton mass matrices is:

$$(\tilde{l}^\dagger_L \tilde{l}^\dagger_R) \begin{pmatrix} m^2_L(1+\delta_{LL}) & (A - \mu \tan\beta) m_l + m_L m_R \delta_{LR} \\ (A - \mu \tan\beta) m_l + m_L m_R {\delta_{LR}}^\dagger & m^2_R(1+\delta_{RR}) \end{pmatrix} \begin{pmatrix} \tilde{l}_L \\ \tilde{l}_R \end{pmatrix}$$

where m_L and m_R are, respectively, the average masses of the L and R sleptons and $A = am_0$ with $a \simeq O(1)$.

The MI now corresponds to a development of the slepton propagators around the diagonal with the average slepton masses, m^2_L and m^2_R. In practice, we are working in the basis of leptons, neutralinos and charginos mass eigenstates and slepton weak eigenstates.

The FV is parametrized through a flavour violating mass insertion in the virtual slepton line. In this basis, the gaugino couplings are flavour-diagonal and so the lagrangian in eq.1 takes the form:

$$\begin{aligned} -\mathcal{L} = {} & \tilde{l}^\dagger_{Li} \overline{\tilde{\chi}^0_A} \Big(N^{A(i)}_{LR} P_R + N^{A(i)}_{LL} P_L \Big) l_i + \tilde{l}^\dagger_{Ri} \overline{\tilde{\chi}^0_A} \Big(N^{A(i)}_{RR} P_R + N^{A(i)}_{RL} P_L \Big) l_i + \\ & + \tilde{\nu}^\dagger_i \overline{\tilde{\chi}^-_A} \Big(C^{A(i)}_{LR} P_R + C^{A(i)}_{LL} P_L \Big) l_i + \text{h.c.} \,, \qquad i = e, \mu, \tau \end{aligned} \tag{3.1}$$

where the coefficient $C_{B,C}^{A(i)}$ and $N_{B,C}^{A(i)}$ (with $B,C=(L,R)$) are:

$$\begin{aligned}
C_{LL}^{A(i)} &= g_2(O_R)_{A1}\,, \qquad C_{LR}^{A(i)} = -\frac{g_1}{\sqrt{2}}\frac{m_{l_i}}{M_W c_\beta}(O_L)_{A2}\\
N_{LL}^{A(i)} &= -\frac{g_2}{\sqrt{2}}(O_N)_{A2} - \frac{g_1}{\sqrt{2}}(O_N)_{A1}\,, \qquad N_{RR}^{A(i)} = \sqrt{2}g_1(O_N)_{A1}\\
N_{LR}^{A(i)} &= N_{RL}^{A(i)} = \frac{g_1}{\sqrt{2}}\frac{m_{l_i}}{M_W c_\beta}(O_N)_{A3}\,,
\end{aligned} \tag{3.2}$$

where $O_{L,R}$ and O_N are hermitian matrices that diagonalize the chargino and neutralino mass matrices, respectively.

3.1 Neutralino-chargino sector: GMI approximation

In the slepton mass matrix, we were able to factorize the source of LFV, namely the mass insertions δ_{ij}.

We would like to have a similar tool in the chargino-neutralino sector too. In fact, it is difficult to understand the dependence of physical quantities on the elements of the chargino-neutralino mass matrices. For instance, in the GMI approximation we treat both the off-diagonal elements (flavour violating or not) of the slepton mass matrices and the off-diagonal terms (flavour conserving) of the chargino and neutralino mass matrices as mass insertions.

In order to understand the validity of this other kind of approximation let us consider the chargino mass matrix M_C:

$$(\bar{\tilde{W}}_R^- \quad \bar{\tilde{H}}_{2R}^-)\begin{pmatrix} M_2 & \sqrt{2}m_W\cos\beta \\ \sqrt{2}m_W\sin\beta & \mu \end{pmatrix}\begin{pmatrix} \tilde{W}_L^- \\ \tilde{H}_{1L}^- \end{pmatrix} + \text{h.c.}\,,$$

diagonalized by 2×2 hermitian matrices O_L and O_R as $O_R M_C O_L^T = (M_C)_{\rm diag}$. Now, assuming that $M_{2,1}$, μ, $|M_{2,1}\pm\mu| \simeq \Lambda \gg M_Z$, we obtain as approximate mass eigenstates $\tilde{W}^-$ and $\tilde{H}^-$ with masses M_2 and μ, respectively. Actually, corrections to the spectrum start from the order $O(M_Z^2/\Lambda^2)$, while the off diagonal elements of the O_R, O_L matrices are of the form $M_W[M_2 s_\beta(c_\beta)+\mu c_\beta(s_\beta)]/[M_2^2-\mu^2]$ where $s_\beta=\sin\beta$ and $c_\beta=\cos\beta$. The approximations made follow naturally from scenarios like m-SUGRA in which a large μ term is required in order to get correct electroweak symmetry breaking (see for instance table II in [17]). The neutralino spectrum can be analyzed in a very similar way. It turns out that a "bino-like" LSP can very easily have the right cosmological abundance to make a good dark matter candidate, so from this point of view the large μ limit may be preferred.

In this approximation the amplitudes of the analyzed processes have two kinds of contributions: one without off-diagonal MI in the gauginos propagator, i.e. pure $\tilde{B}$ and $\tilde{W}$, the other with one MI mixing $\tilde{B}$–$\tilde{H}$, $\tilde{W}$–$\tilde{H}$, proportional to M_W. The approximation consists in keeping at most one flip insertion, only the terms of $\mathcal{O}(M_W/M_i, M_W/\mu)$, so we are neglecting terms of order $\mathcal{O}(M_W^2/m_{\rm susy}^2)$. In this basis both charginos, neutralinos and sleptons are in the flavour-eigenstates and the expressions of the amplitudes are very easy to understand.

4. mSUGRA spectrum

The aim of our analyses is to bound all the leptonic δ_{ij}'s and to establish the limit of applicability of the approximate methods presented above.

In this context, we prefer to work in a well defined model (m-SUGRA) to reduce the number of free parameters and to simplify the physical interpretation of the results.

Let us recall the constraints arising in mSUGRA, where the universality assumption reduces the parameters at the Planck scale to a common scalar mass, m_0, a common gaugino mass, $M_{1/2}$, and a universal $A = am_0$ term with $a \simeq O(1)$.

At the low energy scale, after the RGE running, the parameters M_1, M_2, m_L and m_R are obtained as follows:

$$\begin{aligned}
M_i(m_W) &\simeq \frac{\alpha_i(m_W)}{\alpha_i(M_U)} M_i(M_U) \\
m_L^2(m_W) &\simeq m_L^2(M_U) + 0.5M_2^2(M_U) + 0.04M_1^2(M_U) \\
m_R^2(m_W) &\simeq m_R^2(M_U) + 0.15M_1^2(M_U)
\end{aligned}$$

where M_U is the unification scale and the mSUGRA constraints are satisfied when $M_1(M_U) = M_2(M_U) = M_{1/2}$ and $m_R^2(M_U) = m_L^2(M_U) = m_0^2$.

A very important constraint in mSUGRA comes from the radiative electroweak breaking condition that requires a fine-tuned μ in order to fulfill the minimum condition:

$$|\mu^2| + \frac{M_Z^2}{2} \simeq m_0^2 \frac{1 + 0.5\tan^2\beta}{\tan^2\beta - 1} + M_{1/2}^2 \frac{0.5 + 3.5\tan^2\beta}{\tan^2\beta - 1},$$

so, the spectrum of the model is completely known given the following set of parameters: m_0, $M_{1/2}$, a, $\tan\beta$, $|\mu|$.

5. LFV processes: $l_i \to l_j\gamma$, $l_i \to 3l_j$, $\mu \to e$ in Ti

Let us discuss the branching ratios of the LFV rare processes as $l_i \to l_j\gamma$. The amplitudes of the processes take a form

$$T = m_{l_i}\epsilon^\lambda \overline{u}_j(p-q)[iq^\nu\sigma_{\lambda\nu}(A_L P_L + A_R P_R)]u_i(p)$$

where p and q are momenta of the leptons l_k and of the photon respectively.

The chirality flip of this transition is the reason of the appearance of the m_{l_i} factor in the operator. The branching ratio of $l_i \to l_j\gamma$ is given by

$$\frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)} = \frac{48\pi^3\alpha}{G_F^2}(|A_L^{ij}|^2 + |A_R^{ij}|^2) \sim \frac{\alpha^3}{G_F^2}\frac{\delta_{ij}^2}{\tilde{m}^4}\tan\beta^2$$

with $\tilde{m}$ a typical susy scale. In SUSY, this chirality flip can be implemented in the external fermion line or at Yukawa vertex or in the internal sfermion line through a flavour conserving FC mass insertion $\Delta_{ii}^{RL} = (A - \mu\tan\beta)m_i$.

Each coefficient in the above formula can be written as a sum of two terms,

$$A_{L,R} = A^n_{L,R} + A^c_{L,R}$$

where $A^n_{L,R}$ and $A^c_{L,R}$ stand for the contributions from the neutralino loops and from the chargino loops respectively.

Finally, we consider the $l_i \to 3l_j$ and $\mu \to e$ in Ti processes. Both of them get contributions from penguin-type diagrams (with photon or Z boson exchanges) and from box-type diagrams. However the γ penguin-type contribution is enhanced by a $\tan\beta$ factor with respect to the other contributions so it dominates and one can find the simple theoretical relations:

$$\begin{aligned} Br(l_i \to l_j l_j l_j) &\simeq 7 \times 10^{-3} BR(l_i \to l_j \gamma)\,, \\ Br(\mu \to e \text{ in Ti}) &\simeq 6 \times 10^{-3} BR(\mu \to e\gamma)\,. \end{aligned}$$

Now, imposing the actual experimental upper limits on the above LFV processes, we get the upper bounds on the δ_{ij}'s from each process. In particular, the ratios among the δ_{ij}'s upper bounds from different processes are:

$$\begin{aligned} \delta_{ij}^{l_i \to 3l_j} &\simeq 4\, \delta_{ij}^{l_i \to l_j \gamma} \\ \delta_{21}^{\mu \to e \text{ in Ti}} &\simeq 4 \delta_{21}^{\mu \to e\gamma}\,. \end{aligned}$$

We note that $l_i \to l_j \gamma$ give the more stringent bounds on the δ_{ij}'s. However, these relations are no longer true in special regions, where, strong cancellations reduce $Br(l_i \to l_j\gamma)$ by several order of magnitude and destroy the above relations (see the discussion about δ_{RR} bounds in the next section).

In this paper, we will consider only the contributions from neutralino and chargino sectors. However, Higgs bosons (h^0, H^0, A^0) are also sensitive to flavour violation and mediate processes such as $\mu \to e$ in Nuclei [29], $\tau \to 3\mu$ [30] or $\tau \to \mu\eta$ [31] or $\tau \to \mu(e)\gamma$ [33]. The amplitudes of these processes are sensitive to a higher degree in $\tan\beta$ than the chargino/neutralino ones (the BRs grow as $(\tan\beta)^6$, though they are suppressed by additional Yukawa couplings) and thus could lead to large branching fractions at large $\tan\beta$ [32].

6. Bounds on δ^{ij} from $l_i \to l_j\gamma$, $l_i \to 3l_j$, $\mu \to e$ in nuclei

The approach we follow to put bounds on the various δ^{ij} is to consider only one mass insertion contributing at a time for the $Br(l_i \to l_j\gamma)$. Each off-diagonal FV entry in the slepton mass matrices is put equal to zero, except for the FV insertion we are interested to constrain.

Now, what we mean by full computation is to diagonalize the slepton mass matrix (with only one δ^{ij} term) and to use the experimental $Br(l_i \to l_j\gamma)$ limits to impose constraints on each insertion type.

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Imposing that the contribution of each δ^{ij} does not exceed (in absolute value) the experimental bounds, we obtain the limits on the δ^{ij}'s, barring accidental cancellations.[1]

Hence forward, to be as clear as possible, we will speak in the MI language.

6.1 Bounds on δ_{LL}

The amplitudes proportional to δ_{LL} receive both U(1) and SU(2) type contributions. In the first case we can have a pure $\tilde{B}$ exchange, with chirality-flip implemented in the external (internal) fermion (sfermion) line or at yukawa vertex from $\tilde{B}-\tilde{H}^0$. In the second case we have $\tilde{W}$ and $\tilde{W}-\tilde{H}$ exchange both for charginos and for neutralinos. However in the $\tilde{W}$ case, because the SU(2) Gauge fields don't couple to right-handed fields, we can't make a chirality flip in the internal sfermion line. When the chirality is flipped in the external lepton line, the amplitude does not contain the μ mass term and is not $\tan\beta$ enhanced.

In the following we give the expression for the amplitude $A^{21}_{L2} = (A^{21}_{L2})_{\mathrm{SU}(2)} + (A^{21}_{L2})_{\mathrm{U}(1)}$ of the $l_i \to l_j\gamma$ processes in the GMI approximation:

$$
\begin{aligned}
(A^{21}_{L2})_{\mathrm{SU}(2)} &= \frac{\alpha_2}{4\pi}\Delta^{ij}_{LL}\left[\frac{f_{1n}(a_L)+f_{1c}(a_L)}{m_L^4} + \frac{\mu M_2 t_\beta}{(M_2^2-\mu^2)}\frac{(f_{2n}(a_L,b_L)+f_{2c}(a_L,b_L))}{m_L^4}\right] \\
\left(A^{ij}_L\right)_{\mathrm{U}(1)} &= \frac{\alpha_1}{4\pi}\Delta^{ij}_{LL}\left[\frac{f_{1n}(a_L)}{m_L^4} + \mu M_1 t_\beta\left(\frac{-f_{2n}(a_L,b_L)}{m_L^4(M_1^2-\mu^2)} + \frac{1}{(m_R^2-m_L^2)}\times\right.\right. \\
&\quad\left.\left.\times\left(\frac{2f_{2n}(a_L)}{m_L^4} + \frac{1}{(m_R^2-m_L^2)}\left(\frac{f_{3n}(a_R)}{m_R^2} - \frac{f_{3n}(a_L)}{m_L^2}\right)\right)\right)\right],
\end{aligned}
$$

where $a_L = M_{1,2}^2/m_L^2$ for U(1) or SU(2) contributions, respectively, while $a_R = M_1^2/m_R^2$ and $b_{L,R} = \mu^2/m^2_{L,R}$. The loop functions $f_{i(c,n)}(x)$'s are reported in appendix A while $f_{i(c,n)}(x,y) = f_{i(c,n)}(x) - f_{i(c,n)}(y)$ and $t_\beta = \tan\beta$.

In the above equations, for completeness, we included the subdominant contributions that are not $\tan\beta$ enhanced.

In figure 1 we can see the bounds on δ^{LL}_{21} and δ^{LL}_{32} coming from $\mu\to e\gamma$ and $\tau\to\mu\gamma$, respectively. As for $\tau\to e\gamma$, the limit on δ^{LL}_{31} is such that $\delta^{LL}_{31}/\delta^{LL}_{32} = (Br(\tau\to e\gamma)/Br(\tau\to\mu\gamma))^{1/2}$. The red lines correspond to the full computations while the green and the blue lines refer to the mass insertion (MI) and to the generalized mass insertion (GMI) approximations, respectively.

In the δ^{LL}_{21} case, MI and full computations give indistinguishable results and the GMI gives a very satisfactory approximation (see the next section for a quantitative estimate). In the δ^{LL}_{32} case the degree of approximation is worse than in the δ^{LL}_{21} case. The motivation is that the flavour violating (FV) and conserving (FC) insertions of the 32 sector are generally larger than those relative to the 21 sector. For instance, $\delta^{LL}_{32} \simeq 10^2\delta^{LL}_{21}$ and $\delta^{RL}_{33} = m_\tau/m_\mu\delta^{RL}_{22}$.

A large FV insertion, as for $\delta^{LL}_{32} = 0.3$, produces a sizable distinction between approximated and full computations as it must be when we go away from the perturbative region.

[1]In SUSY GUT theories there are possible correlations between the hadronic and leptonic FV effects [34, 35].

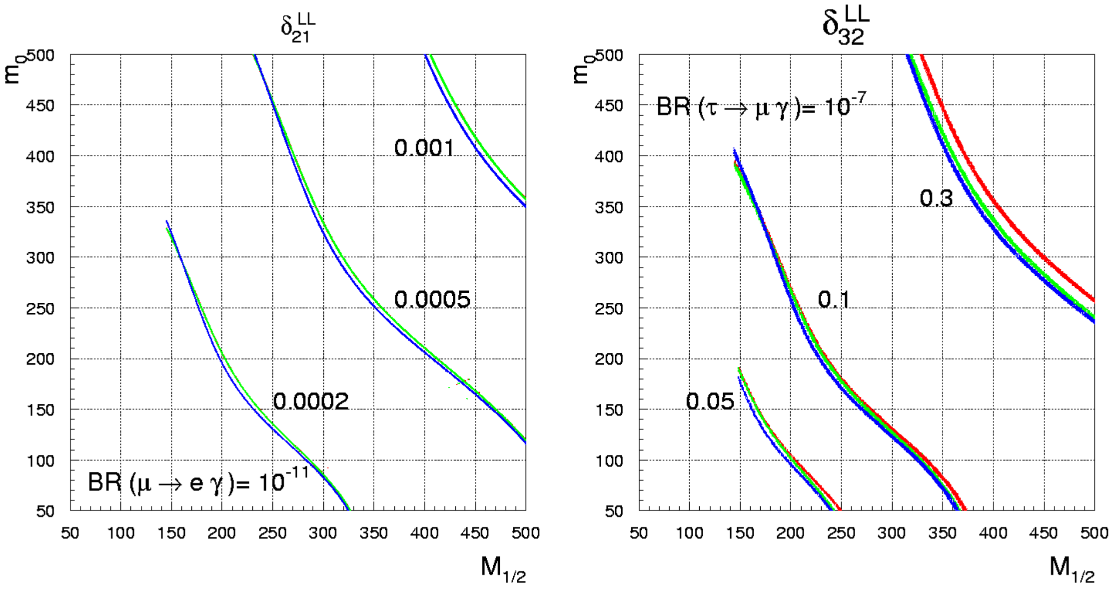

Figure 1: Upper limits on δ_{LL}^{ij} from $l_i \to l_j\gamma$. In the plots we set $\mu > 0$, $\tan\beta = 10$ and $a = 0$. Red lines correspond to full computation, green and blue lines to MI and GMI approximations respectively.

The effect of a large FC insertion is evident for small m_0, where the μ term is much larger than the slepton masses, so that to treat $m_\tau \mu \tan\beta/\tilde{m}_{L,R}^2$ as insertion is not properly correct.

In figure 1, the most appreciable deviations between the full computation and the GMI one are due to the approximations in the neutralino mixing and they tend to vanish in the large $\tan\beta$ limit, where the neutralino (chargino) mass matrix expansion is better justified.

We remark that we don't see these deviations in the regions with chargino or pure-Bino dominance (in fact these last contributions are lesser affected by the GMI approximations). The bounds are rather insensitive to the sign of μ.

6.2 Bounds on δ_{LR}

In this case, the only contribution arises from the $\tilde{B}$ exchange and the amplitude does not contain a $\tan\beta$ factor so δ_{LR}^{ij} bound is $\tan\beta$ independent, unlike all the other bounds.

In the GMI approximation, the amplitude has the following expression:

$$A_{L1}^{ij} = \frac{\alpha_1}{4\pi} \frac{\Delta_{RL}^{ij}}{(m_L^2 - m_R^2)} \left(\frac{M_1}{m_{l_i}}\right) \left(\frac{f_{3n}(a_R)}{m_R^2} - \frac{f_{3n}(a_L)}{m_L^2}\right).$$

We note that the chirality flip is realized directly by the mass insertion so we can understand the order of the bound, compared to the LL case, $\delta_{ij}^{LR} \simeq (m_i/\tilde{m}) \tan\beta \delta_{ij}^{LL}$, as confirmed numerically.

While MI and full computations give practically the same result both in 21 and in 32 sectors, the GMI approximation starts to work very well when $M_{1/2} \geq 300\,\mathrm{GeV}$. This

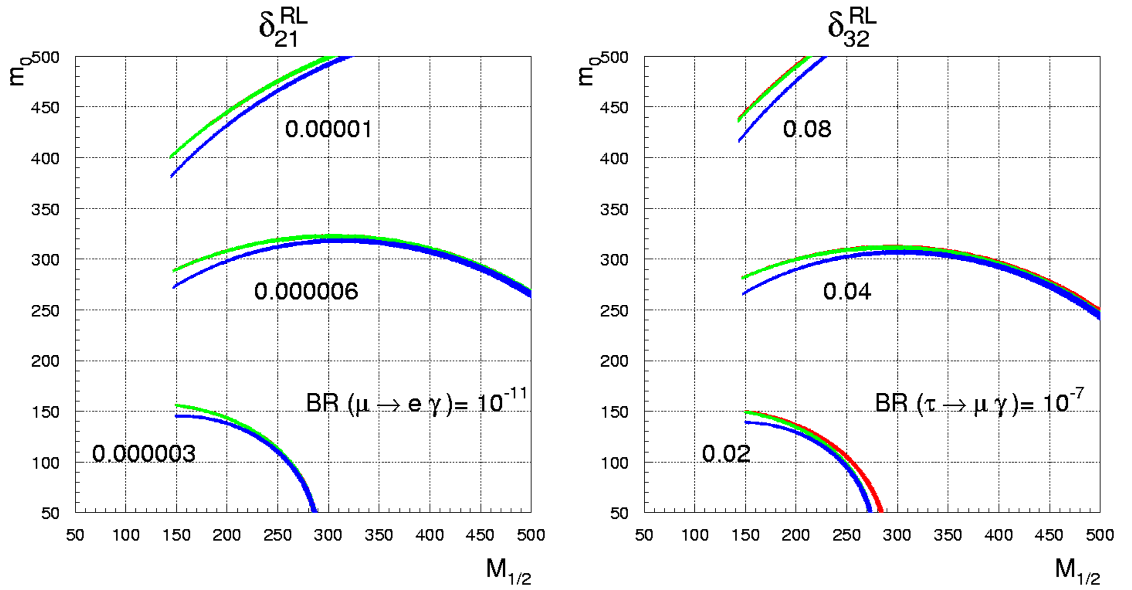

Figure 2: Upper limits on δ_{RL}^{ij} from $l_i \to l_j\gamma$. In the plots we set $\mu > 0$, $\tan\beta = 10$ and $a = 0$. Red lines correspond to full computation, green and blue lines to MI and GMI approximations respectively.

result can be better understood by bearing in mind the conditions under which the GMI approximation can be applied. A necessary condition is that $M_{1,2} \geq m_W$ which is not satisfied when $M_{1/2} \leq 300\,\text{GeV}$.

In the LL case we did not have this problem because of the chargino dominance. We remind that $M_2 \simeq 2M_1$ so we would expect the same behavior when $M_{1/2} \leq 150\,\text{GeV}$ but this region is forbidden by the LSP Bino constraint.

In the 32 sector, both MI and GMI approximations show a sizable deviation from the full computation in small m_0 regions because of a large δ_{RL}^{33}.

A large δ_{RL}^{33} induces a mass split of order δ_{RL}^{33} in the third generation so, in the mass insertion language, we are neglecting next to leading terms of order $\mathcal{O}((\delta_{RL}^{33})^2)$ not so suppressed in the examined case. We note that the same argument holds for the δ_{LL}^{32} case.

6.3 Bounds on δ_{RR}

The δ_{RR} sector requires a bit more of attention because of some cancellations occurring among the amplitudes in some regions of the parameter space.

The bounds on δ_{RR} become very weak so, it is interesting to check what is limit of applicability of the mass insertion approximations.[2]

The origin of this cancellations is the destructive interference between the dominant contributions coming from the $\tilde{B}$ (with chirality flip implemented through a FC mass insertion) and $\tilde{B}\tilde{H}^0$ exchange. To better understand the nature of these cancellations let

[2]This typically occurs for RR type MI as long as universality in the gaugino masses is maintained at the high scale. Although in a completely generic situation without any universal boundary conditions, such cancellations can occur for LL type MI also [36].

us derive, in the GMI approximation, the amplitude associated to δ_{RR}:

$$A_R^{ij} = \frac{\alpha_1}{4\pi}\Delta_{RR}^{ij}\Bigg[\frac{4f_{1n}(a_R)}{m_R^4} + \mu M_1 t_\beta\bigg(\frac{2f_{2n}(a_R,b_R)}{m_R^4(M_1^2-\mu^2)} + \frac{1}{(m_L^2-m_R^2)}\times \times\bigg(\frac{2f_{2n}(a_R)}{m_R^4} + \frac{1}{(m_L^2-m_R^2)}\bigg(\frac{f_{3n}(a_L)}{m_L^2} - \frac{f_{3n}(a_R)}{m_R^2}\bigg)\bigg)\bigg)\Bigg].$$

In the above equation, the first term is relative to the pure Bino amplitude with external chirality flip, the second one corresponds to the $\tilde{B}\tilde{H}^0$ mixing in the neutralino sector while the last term originates from the $\tilde{B}$ contribution with internal sfermion line chirality flip.

It is easy to check numerically that the dominant contributions, proportional to $\tan\beta$, have opposite sign in all the parameter space. To get a feeling of the reason of the above opposite sign, we note that the amplitude relative to $\tilde{B}$ (with chirality flip implemented through a FC mass insertion) is proportional to $N_{LL}N_{RR}$ with $sign(N_{LL}N_{RR}) = -1$ while the contribution arising from the $\tilde{B}\tilde{H}^0$ exchange is proportional to $N_{LR}N_{RR}$ with $sign(N_{LR}N_{RR}) = +1$ (see eqs.3). Vice-versa, the same type of contributions in the δ_{LL} case have the same sign being proportional to $N_{RR}N_{LL}$ with $sign(N_{RR}N_{LL}) = -1$ and to $N_{RL}N_{LL}$ with $sign(N_{RL}N_{LL}) = -1$, respectively. The above difference depends on the opposite sign between the hypercharge of SU(2) doublets and U(1) singlets.

If some cancellations occur among the leading contributions, subleading effects, generally disregarded, could become important or even dominant.

In this spirit, we retain the amplitude relative to a chirality flip realized in the external fermion line, neglected in [14] because it is not $\tan\beta$ enhanced.

Moreover, the above amplitude shows that while the dominant ($\tan\beta$ enhanced) contributions are proportional to the μ mass term[3] the pure B amplitude with external chirality flip is μ independent. In this way, the branching ratio is not invariant under the change of the μ sign so, in general, one has to consider both cases.

In figure 3 we show the upper limits on δ_{RR}^{21} from $\mu \to e\gamma$. The limit on δ_{3j}^{RR} are simply obtained by $\delta_{3j}^{RR}/\delta_{21}^{RR} = (Br(\tau \to l_j\gamma)/Br(\mu \to e\gamma))^{1/2}$ thus, by now δ_{32}^{RR} and δ_{31}^{RR} are not constrained at all.

As we can see, we are not able to remove these cancellations, in fact, the effect of the $\tan\beta$ independent contribution is only a shift of the cancellation region (the same thing happens if we flip the μ sign).

We remark that, such cancellations occur in all the situations where the contribution of the A term to $\delta_{RL}^{22} = (A - \mu\tan\beta)m_\mu$ is negligible. There are well known model dependent upper bounds on the A parameter to avoid color and e.m. charge breaking, in particular $|A|/m_R \leq 3$ in mSUGRA. Moreover, mSUGRA requires a large μ term to fulfill the electroweak symmetry breaking, thus, we cannot invert the relative sign between the two amplitudes. In this spirit, we neglected the A term in A_R^{ij}. It is noteworthy that the MI approximation works very well reproducing the same cancellation regions as the full computation. In the GMI case, we have a net shift of this region but the general structure is maintained.

[3] In reality, this is true only if we neglect the A term in $\delta_{RL}^{22} = (A - \mu\tan\beta)m_\mu$ in the pure B amplitude.

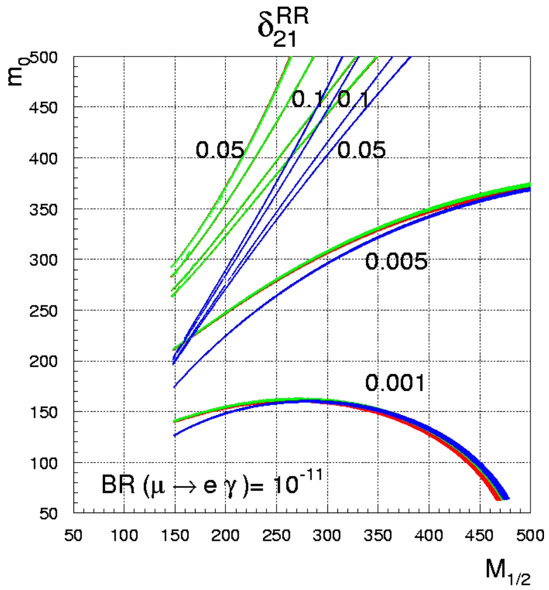

Figure 3: Upper limits on δ^{21}_{RR} from $\mu \to e\gamma$; In the plots we set $\mu > 0$, $\tan\beta = 10$ and $a = 0$ and red lines correspond to the full computation, green and blue lines to the MI and the GMI approximations, respectively.

In conclusion, $\mu \to e\gamma$ doesn't allow to put a bound in the RR sector, so, we take into account other LFV processes as $\mu \to eee$ and $\mu \to e$ in Nuclei.

As figure 5 shows, we find that the last process suffers from a bigger cancellation problem than $\mu \to e\gamma$ (in the RR sector) while $\mu \to eee$ does not.

This result requires some explanations. As we have seen in section 5, the dominant contribution to $Br(\mu \to e$ in Nuclei) and $Br(\mu \to eee)$ arises from the dipole operator (that is $\tan\beta$ enhanced) but, if the dipole amplitude is strongly suppressed by some cancellations, $Br(\mu \to e\gamma)$ goes to zero while $Br(\mu \to e$ in Nuclei) and $Br(\mu \to eee)$ are dominated by non-dipole contributions. So, in principle, $\mu \to e$ in Nuclei and $\mu \to eee$ could be able to bound δ_{RR}. As we can see in figure 5, this is the case of $\mu \to eee$ that gives a bound for $\delta_{RR} \leq 0.4$ that is, correctly, $\tan\beta$ independent.

On the other hand, we find that $Br(\mu \to e$ in Nuclei) has additional cancellations between dipole and not-dipole amplitudes. As a final effect, we have that $\mu \to e$ in Nuclei suffers from a similar cancellation problem as $\mu \to e\gamma$ but, as can be expected, in a different region of the parameter space.

Because of this strong cancellations, $\mu \to e\gamma$ and $\mu \to e$ in Nuclei prevent us from getting a bound in the RR sector both at the present and even in the future when their experimental sensitivity will be improved.

However, an interesting feature is that $\mu \to e\gamma$ and $\mu \to e$ in Nuclei amplitudes have cancellations in different regions, so, if we combine the two processes, we obtain a more stringent bound ($\delta^{RR}_{21} \leq 0.2$) than the one coming from $\mu \to eee$ ($\delta^{RR}_{21} \leq 0.4$). It is noteworthy that the study of combined processes allows to extract additional informations respect to each separate case.

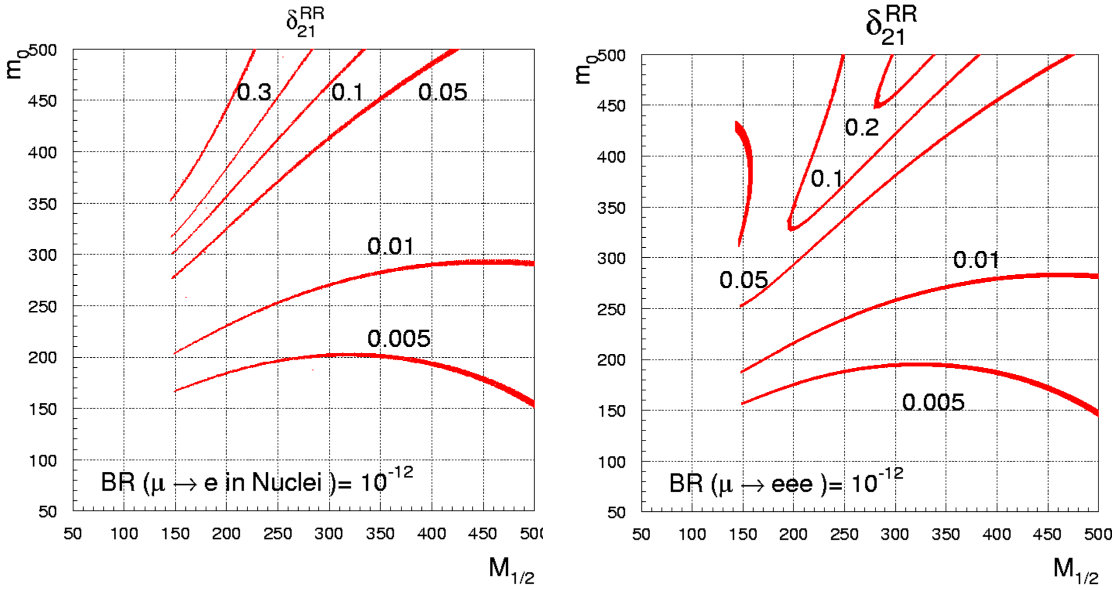

Figure 4: Upper limits on δ_{RR}^{21} from $\mu \to e$ in Ti (a) and $\mu \to eee$ (b). In the plots we set $\mu > 0$, $\tan\beta = 10$ and $a = 0$.

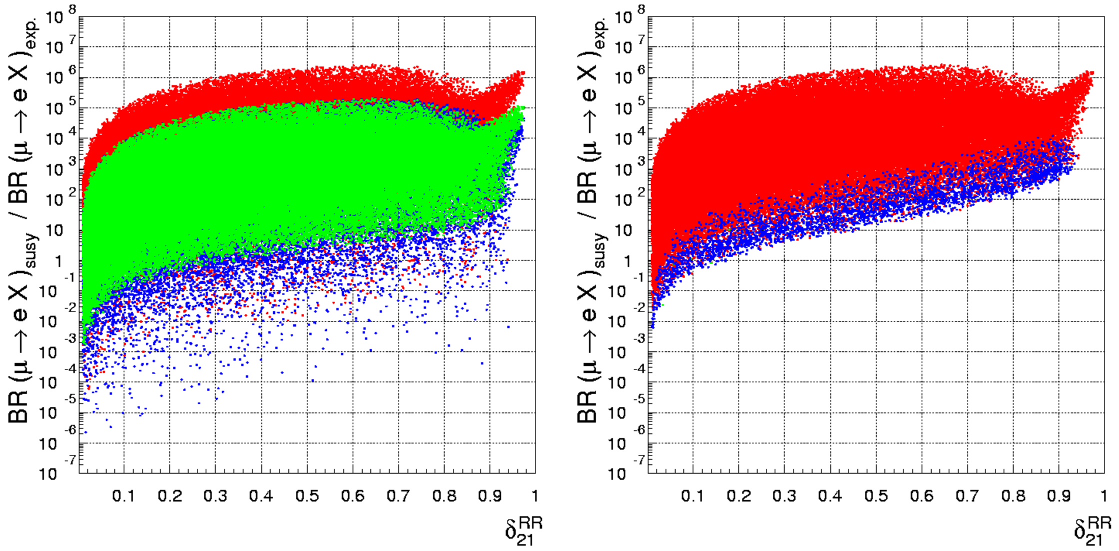

Figure 5: Branching ratios of $\mu \to e$ transitions normalized to the actual experimental upper bounds vs δ_{RR}^{21}. Red dots correspond to $X = \gamma$, blue dots to $X =$ "in Nuclei" and green dots to $X = ee$. In the scatter plots we have taken $\mu > 0$, $3 < \tan\beta < 40$ $50\,\text{GeV} \leq m_0 \leq 500\,\text{GeV}$, $50\,\text{GeV} \leq M_{1/2} \leq 500\,\text{GeV}$ and $-3 < a < 3$. On the right, for each point of the parameter space, it is shown only the biggest normalized Br value among $\mu \to e\gamma$, $\mu \to eee$ and $\mu \to e$ in Ti.

6.4 Bounds on the double mass insertion δ^{32} δ^{31} from $\mu \to e\gamma$

As we have seen in the previous section, the bounds on $\tau \to \mu\gamma$ and $\tau \to \mathrm{e}\gamma$ are very loose. This is due to a worse experimental resolution on the above processes compared to the $\mu \to \mathrm{e}\gamma$ one. However, we can extract additional information in the 32 and 31 sectors applying to $\mu \to e\gamma$.

The point is that we are able to put bounds on the product of two mass insertions, namely $\delta_{32}\delta_{31}$. In general, we can pass from the second to the first generation or through the δ_{21} or through the $\delta_{23}\delta_{31}$ insertions.

Now, to constraint the MIs, we proceed exactly as for a single MI. We put to zero all off diagonal FV entries in the slepton mass matrix except for the two MIs we are interested to bound. At this point, we give the analytical expressions for all the $\delta_{23}\delta_{31}$ type insertions in the GMI approach.

For $\delta_{23}^{RL}\delta_{31}^{LL}$ we get the following amplitude:

$$A_{L1}^{21} = \frac{\alpha_1}{4\pi}\frac{M_1}{m_\mu}\frac{\Delta_{RL}^{23}\Delta_{LL}^{31}}{(m_L^2-m_R^2)}\left[\frac{2f_{2n}(a_L)}{m_L^4}+\frac{1}{(m_R^2-m_L^2)}\left(\frac{f_{3n}(a_R)}{m_R^2}-\frac{f_{3n}(a_L)}{m_L^2}\right)\right].$$

The amplitude A_{R1}^{21}, relative to $\delta_{23}^{LR}\delta_{31}^{RR}$, is obtained by $A_{R1}^{21} = A_{L1}^{21}(L\leftrightarrow R)$. In figure 6 we show the bounds on $\delta_{23}^{RL}\delta_{31}^{LL}$ and on $\delta_{23}^{LR}\delta_{31}^{RR}$.

As we can see, they exhibit different behaviors, especially for m_0 smaller than $M_{1/2}$ due to the m_R and m_L mass difference (in fact, while in the first case we have two left handed and one right handed sfermions running in the loop, in the second case we have the opposite situation).

So, while δ_{RL}^{ij} and δ_{LR}^{ij} are indistinguishable, it is not so for $\delta_{23}^{LR}\delta_{31}^{RR}$ and $\delta_{23}^{RL}\delta_{31}^{LL}$.

The amplitude associated to $\delta_{23}^{LL}\delta_{31}^{LL}$, namely $A_{L2}^{21} = (A_{L2}^{21})_{\mathrm{SU(2)}} + (A_{L2}^{21})_{\mathrm{U(1)}}$, reads:

$$\begin{aligned}
(A_{L2}^{21})_{\mathrm{SU(2)}} &= \frac{\alpha_2}{4\pi}\Delta_{LL}^{23}\Delta_{LL}^{31}\left[\frac{I_{1n}(a_L)+I_{1c}(a_L)}{m_L^6}+\frac{\mu M_2 t_\beta}{(M_2^2-\mu^2)}\times\right.\\
&\qquad\left.\times\frac{I_{2n}(a_L,b_L)+I_{2c}(a_L,b_L)}{m_L^6}\right]\\
(A_{L2}^{21})_{\mathrm{U(1)}} &= \frac{\alpha_1}{4\pi}\Delta_{LL}^{23}\Delta_{LL}^{31}\left[\frac{I_{1n}(a_L)}{m_L^6}+\mu M_1 t_\beta\left(\frac{-I_{2n}(a_L,b_L)}{m_L^6(M_1^2-\mu^2)}+\frac{1}{(m_R^2-m_L^2)}\times\right.\right.\\
&\qquad\times\left(\frac{2I_{2n}(a_L)}{m_L^6}+\frac{2f_{2n}(a_L)}{m_L^4(m_R^2-m_L^2)}+\right.\\
&\qquad\left.\left.\left.+\frac{1}{(m_R^2-m_L^2)^2}\left(\frac{f_{3n}(a_R)}{m_R^2}-\frac{f_{3n}(a_L)}{m_L^2}\right)\right)\right)\right].
\end{aligned} \tag{6.1}$$

The contribution arising from a $\delta_{LL}^{23}\delta_{RR}^{31}$ type insertion reads:

$$\begin{aligned}
A_{L3}^{21} &= -2\frac{\alpha_1}{4\pi}\mu M_1 t_\beta\frac{\Delta_{LL}^{23}\Delta_{RR}^{31}}{(m_L^2-m_R^2)^2}\left[\frac{f_{2n}(a_L)}{m_L^4}+\frac{f_{2n}(a_R)}{m_R^4}+\right.\\
&\qquad\left.+\frac{1}{(m_R^2-m_L^2)}\left(\frac{f_{3n}(a_R)}{m_R^2}-\frac{f_{3n}(a_L)}{m_L^2}\right)\right].
\end{aligned}$$

In the figure 7 we show the bounds for $\delta_{23}^{LL}\delta_{31}^{LL}$ and for $\delta_{23}^{LL}\delta_{31}^{RR}$ (equal to the bounds on $\delta_{23}^{RR}\delta_{31}^{LL}$). It is to note that $\delta_{23}^{LL}\delta_{31}^{RR}$ is strongly constrained because, the associate amplitude, is m_τ/m_μ enhanced respect to the usual Bino-like mediated processes (being the chirality flip implemented in the internal sfermion line through δ_{33}^{LR} and not by δ_{22}^{LR}, as usual). The amplitude A_{R3}^{21}, relative to $\delta_{23}^{RR}\delta_{31}^{LL}$, is $A_{R3}^{21} = A_{L3}^{21}(L\leftrightarrow R)$.

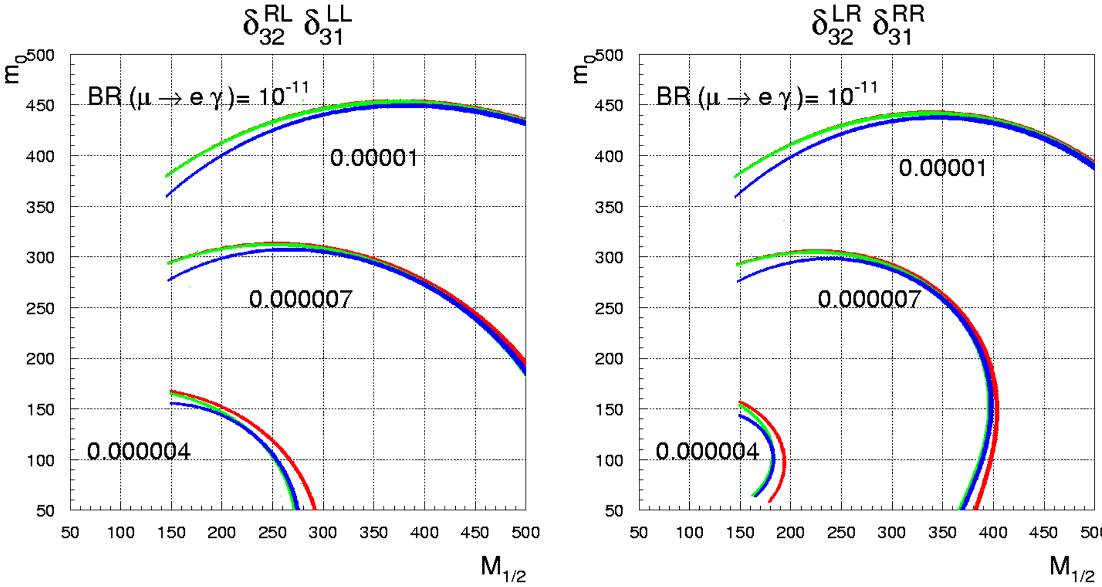

Figure 6: Upper limits on δ^{32}_{RL} δ^{31}_{LL} and δ^{32}_{LR} δ^{31}_{RR} from $\mu \to e\gamma$. We have set $\mu > 0$, $\tan\beta = 10$ and $a = 0$. Red lines correspond to full computation, green and blue lines to MI and GMI approximations respectively.

Finally we derive the expression for the amplitude associated to $\delta^{RR}_{23}\delta^{RR}_{31}$:

$$\begin{aligned}(A^{21}_{R2}) = \frac{\alpha_1}{4\pi}\Delta^{23}_{RR}\Delta^{31}_{RR}\Bigg[\frac{4I_{1n}(a_R)}{m_R^6} + \mu M_1 t_\beta\Bigg(&\frac{2I_{2n}(a_R,b_R)}{m_R^6(M_1^2-\mu^2)} + \frac{1}{(m_R^2-m_L^2)}\times \\ &\times\left(\frac{-2I_{2n}(a_R)}{m_R^6} + \frac{2f_{2n}(a_R)}{m_R^4(m_R^2-m_L^2)} + \right. \\ &\left. + \frac{1}{(m_R^2-m_L^2)^2}\left(\frac{f_{3n}(a_R)}{m_R^2} - \frac{f_{3n}(a_L)}{m_L^2}\right)\right)\Bigg)\Bigg].\end{aligned}$$

We note that, in general, $\delta_{23}\delta_{31}$ has a bound comparable to δ_{21}, being $\delta_{21} \simeq \delta_{23}\delta_{31}$. The only exception is for $\delta^{RR}_{23}\delta^{LL}_{31}$ ($\delta^{LL}_{23}\delta^{RR}_{31}$) as discussed above.

7. Mass eigenstate vs mass insertion: a numerical analysis

At this point, our purpose is to make a quantitative estimate of the goodness of the MI and GMI approximations.

Even if we have already seen that the above approximations give us the same bounds as the full computation, we were not able, in the previous analyses, to quantify and to distinguish correctly the approximations induced by the chargino (neutralino) branch and by the slepton branch. In the slepton case, we can distinguish between two sources of approximations, namely, FC and FV terms.

In figure 8 we show the ratios between full/MI computations (red dots) and full/GMI computations (blue dots) vs. FV mass insertions.

As we can see in the 21 sector, MI approximates correctly (at $(5-10)\%$ level) the full computation until large FV insertions ($\delta_{21} \simeq 0.2$), while the two computations are prac-

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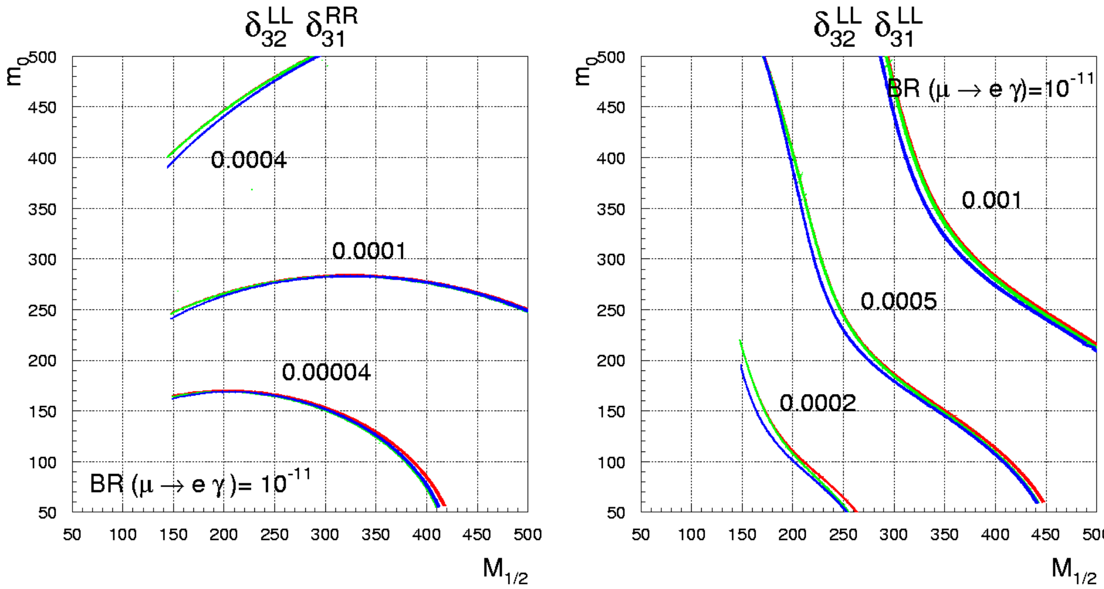

Figure 7: Upper limits on δ_{LL}^{32} δ_{RR}^{31} and δ_{LL}^{32} δ_{LL}^{31} from $\mu \to e\gamma$. We have set $\mu > 0$, $\tan\beta = 10$ and $a = 0$. Red lines correspond to the full computation, green and blue lines to the MI and to the GMI approximations respectively.

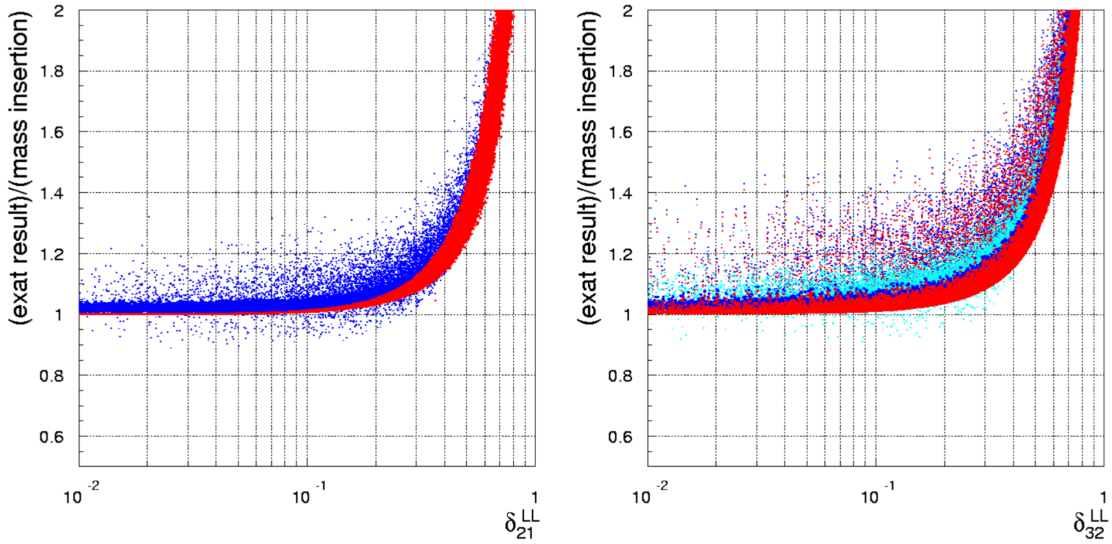

Figure 8: Ratios between exact and MI approximation (red dots) and between exact and GMI approximation (blue dots) vs δ_{LL}^{ij}. On the right, tiny light blue dots are relative to the chargino-neutralino approximations while tiny blue and red dots are due to large FC terms effects.

tically indistinguishable for $\delta_{21} \leq 0.1$. The induced approximations are easily understood if we remind that the approach we followed to put the bounds on the δ^{ij} insertions was to consider only one mass insertion contributing at a time for the $Br(l_i \to l_j\gamma)$. If we stop in the first term we neglect terms of order δ_{ij}^3 in the amplitudes inducing, naively, an approximation on the branching ratios $(Br(\delta_{ij} + \delta_{ij}^3) - Br(\delta_{ij}))/Br(\delta_{ij} + \delta_{ij}^3) \sim 2\delta_{ij}^2$, as it is well reproduced numerically. The FC insertion δ_{22}^{LR} does not produce any sizable effects, in fact, in the worse case (for large μ and $\tan\beta$ and moderate slepton masses), we have $\delta_{22}^{LR} \leq 0.1$. The last argument is not true in the 32 sector where, being $\delta_{33}^{LR}/\delta_{22}^{LR} \simeq m_\tau/m_\mu$, we can have a not perturbative FC insertion. This is clear in figure 8 where, the MI and the GMI approaches underline a $(40-50)\%$ deviations respect to the exact calculation (tiny dots refer to the $\tan\beta \geq 30$ and $M_{1/2} \geq 300$, region where we have sizable deviations due to the slepton FC terms only).

Now, we want to discuss the approximations brought from the chargino and the neutralino sectors in the GMI approach (the following discussion is obviously flavour independent).

As discussed in section 3, it is allowed to use this method when the elements outside from the diagonal (proportional to m_W) are much smaller than those diagonals (μ and $M_{2,1}$).

On the left of figure 8, tiny blue dots show a $(20-30)\%$ approximation (it happens in $M_{1/2} \leq 300$ and $\tan\beta \leq 10$) while the much larger ones refer to the $M_{1/2} \geq 300$ and $\tan\beta \geq 10$ region where the GMI conditions are fulfilled. In the 32 case light blue dots correspond to deviations induced by the GMI approximation in fact they are related to a range of parameters ($m_0 \geq 300$ and $\tan\beta \leq 5$) where $\delta_{33}^{LR} \leq 0.1$.

To summarize the results found, we can say that MI approximation produce the same features as the exact calculation even if strong cancellations occur.

The approach works better in the 21 sector than in the 32 one because, while in the first case we always have perturbative FC terms, in the second case it is not so and we can reach sizable deviations (up to a $(40-50)\%$ level) from the full computation.

Moreover, we have a 10% approximation until FV terms of order 0.2.

The GMI approximation works very well, as the MI approximation, up to gaugino masses heavier than 150 GeV and it produces the cancellations in a shifted region respect to the exact case.

In conclusion we can say that, except for fine-tuned cases, the last approach is very satisfactory.

8. Conclusions

In this work, in the first stage, we studied the constraints on flavour violating terms in low energy SUSY coming from $l_i \to l_j\gamma$.

We have carried out the analysis both in the mass eigenstate and in the mass insertion approximations clarifying the limit of applicability of these approximations.

In particular, we focused on the RR sector where strong cancellations make the sector unconstrained.

We showed that these cancellations prevent us from getting a bound in the RR sector both at the present and even in the future when the experimental sensitivity on $Br(l_i \to l_j\gamma)$ will be improved.

Finally we took into consideration the bounds on the various double mass insertions: $\delta_{23}^{LL}\delta_{31}^{LL}$, $\delta_{23}^{RR}\delta_{31}^{RR}$, $\delta_{23}^{LR}\delta_{31}^{RR}$, $\delta_{23}^{RL}\delta_{31}^{LL}$ and $\delta_{23}^{RR}\delta_{31}^{LL}$. It is clear that the bounds are approximatively the same as in δ_{21} being $\delta_{21} \simeq \delta_{23}\delta_{31}$ except for the last one suppressed by a m_μ/m_τ

factor. So, in spite of very weak bounds on δ_{32} and on δ_{31} coming from $Br(\tau \to \mu\gamma)$ and $Br(\tau \to e\gamma)$ respectively, we have stronger bounds on their product thanks to $Br(\mu \to e\gamma)$ experimental sensitivity. These limits are important in order to get the largest amount of information on SUSY flavour symmetry breaking.

Summarizing the results found in this first stage, we can say that MI approximation produce the same features as the exact calculation even if strong cancellations occur.

The approach works better in the 21 sector, up to $(5-10)\%$ approximation level until $\delta_{21} \leq 0.2$, than in the 32 sector, where, large off diagonal flavour conserving terms can induce a $(40-50)\%$ deviation from the full computation.

The GMI approximation works very well, as the MI approximation, except for some special regions where induces a $(20-30)\%$ approximation with respect to the exact calculation and produces the cancellations in a shift region respect to the exact case. In conclusion, we can say that, except for fine-tuned cases, the last approach is very satisfactory.

In a second stage, being our aim to find constraints in the RR sector, we examined other LFV processes as $\mu \to eee$ and $\mu \to e$ in Nuclei.

We found that the last process suffers from a bigger cancellation problem than $\mu \to e\gamma$ (in the RR sector) while $\mu \to eee$ does not.

However, an interesting feature is that $\mu \to e\gamma$ and $\mu \to e$ in Nuclei amplitudes have cancellations in different regions, so, if we combine the two processes, we obtain a more stringent bound ($\delta_{21}^{RR} \leq 0.2$) than the one coming from $\mu \to eee$ ($\delta_{21}^{RR} \leq 0.4$). It is noteworthy that the study of combined processes allows to extract additional information respect to an individual analysis of all these processes. In particular, it makes it possible to put bounds on sectors previously unconstrained by $\mu \to e\gamma$.

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A. Loop functions

In this appendix we report the explicit expressions for the loop functions appearing in the text:

$$
\begin{aligned}
f_{1n}(a) &= \frac{-17a^3 + 9a^2 + 9a - 1 + 6a^2(a+3)\ln a}{24(1-a)^5} \\
f_{2n}(a) &= \frac{-5a^2 + 4a + 1 + 2a(a+2)\ln a}{4(1-a)^4} \\
f_{3n}(a) &= \frac{1 + 2a\ln a - a^2}{2(1-a)^3} \\
f_{1c}(a) &= \frac{-a^3 - 9a^2 + 9a + 1 + 6a(a+1)\ln a}{6(1-a)^5}
\end{aligned}
$$

$$
\begin{aligned}
f_{2c}(a) &= \frac{-a^2-4a+5+2(2a+1)\ln a}{2(1-a)^4} \\
I_{1n}(a) &= \frac{3a^4+44a^3-36a^2-12a+1-12a^2(2a+3)\ln a}{24(1-a)^6} \\
I_{2n}(a) &= \frac{a^3+9a^2-9a-1-6a(a+1)\ln a}{4(1-a)^5} \\
I_{1c}(a) &= \frac{10a^3+9a^2-18a-1-3a(3+6a+a^2)\ln a}{6(1-a)^6} \\
I_{2c}(a) &= \frac{3a^2-3-(a^2+4a+1)\ln a}{(1-a)^5}\,.
\end{aligned}
$$

References

[1] T.P. Cheng and L.-F. Li, *$\mu \to e\gamma$ in theories with Dirac and Majorana neutrino mass terms*, *Phys. Rev. Lett.* **45** (1980) 1908;
G. Altarelli, L. Baulieu, N. Cabibbo, L. Maiani and R. Petronzio, *Muon number nonconserving processes in gauge theories of weak interactions*, *Nucl. Phys.* **B 125** (1977) 285;
S.T. Petcov, *The processes $\mu \to e+\gamma$, $\mu \to e+e+\bar{e}$, $\nu' \to \nu+\gamma$ in the Weinberg-Salam model with neutrino mixing*, *Sov. J. Nucl. Phys.* **25** (1977) 340;
W.J. Marciano and A.I. Sanda, *Exotic decays of the muon and heavy leptons in gauge theories*, *Phys. Lett.* **B 67** (1977) 303.

[2] For a recent review, see A. Masiero, S.K. Vempati and O. Vives, *Massive neutrinos and flavour violation*, *New J. Phys.* **6** (2004) 202 [hep-ph/0407325], and references therein.

[3] P. Minkowski, *mu → e gamma at a rate of one out of 1-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421;
T. Yanagida, in *Proc. Workshop on Unified Theories & C.*, O. Sawada and A. Sugamoto eds., Tsukuba, Feb 1979;
M. Gell-Mann, P. Ramond and R. Slansky, *The family group in grand unified theories*, in *Sanibel talk*, CALT-68-709 Feb 1979 [hep-ph/98090459];
in *Supergravity*, North Holland, Amsterdam 1979 p. 315;
S.L. Glashow, in *Quarks and leptons*, Cargèse 1979, M. Lévy et al. eds., Plenum Press, New York 1980, p. 707;
R.N. Mohapatra and G. Senjanovic, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.

[4] F. Borzumati and A. Masiero, *Large muon and electron number violations in supergravity theories*, *Phys. Rev. Lett.* **57** (1986) 961.

[5] MEGA collaboration, M.L. Brooks et al., *New limit for the family-number non-conserving decay $\mu^+ \to e^+\gamma$*, *Phys. Rev. Lett.* **83** (1999) 1521 [hep-ex/9905013].

[6] Belle collaboration, K. Abe et al., *An upper bound on the decay $\tau \to \mu\gamma$ from Belle*, *Phys. Rev. Lett.* **92** (2004) 171802 [hep-ex/0310029].

[7] BELLE collaboration, Y. Yusa, H. Hayashii, T. Nagamine and A. Yamaguchi, *Search for neutrinoless τ decays $\tau \to 3\ell$ and $\tau \to \ell K^0_S$*, *eConf* **C0209101** (2002) TU13 [hep-ex/0211017];

Belle collaboration, Y. Yusa et al., *Search for neutrinoless decays $\tau \to 3\ell$*, *Phys. Lett.* **B 589** (2004) 103 [hep-ex/0403039].

[8] BABAR collaboration, B. Aubert et al., *Search for lepton flavor violation in the decay $\tau^- \to \ell^- \ell^+ \ell^-$*, *Phys. Rev. Lett.* **92** (2004) 121801 [hep-ex/0312027];
BABAR collaboration, B. Aubert et al., *Search for lepton flavor violation in the decay $\tau \to \mu\gamma$*, *Phys. Rev. Lett.* **95** (2005) 041802 [hep-ex/0502032].

[9] Belle colloboration, K. Inami, talk presented at the *19th international workshop on weak interactions and neutrinos (WIN-03)* October 6–11, 2003, Lake Geneva, Wisconsin, USA.

[10] Web page: http://meg.psi.ch.

[11] L.J. Hall, V.A. Kostelecky and S. Raby, *New flavor violations in supergravity models*, *Nucl. Phys.* **B 267** (1986) 415.

[12] F. Gabbiani and A. Masiero, *Fcnc in generalized supersymmetric theories*, *Nucl. Phys.* **B 322** (1989) 235.

[13] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A complete analysis of fcnc and CP constraints in general SUSY extensions of the Standard Model*, *Nucl. Phys.* **B 477** (1996) 321 [hep-ph/9604387].

[14] I. Masina and C.A. Savoy, *Sleptonarium (constraints on the CP and flavour pattern of scalar lepton masses)*, *Nucl. Phys.* **B 661** (2003) 365 [hep-ph/0211283].

[15] S. Pokorski, J. Rosiek and C.A. Savoy, *Constraints on phases of supersymmetric flavour conserving couplings*, *Nucl. Phys.* **B 570** (2000) 81 [hep-ph/9906206].

[16] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Lepton-flavor violation via right-handed neutrino Yukawa couplings in supersymmetric Standard Model*, *Phys. Rev.* **D 53** (1996) 2442 [hep-ph/9510309];
J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Exact event rates of lepton flavor violating processes in supersymmetric* SU(5) *model*, *Phys. Lett.* **B 391** (1997) 341 [hep-ph/9605296], erratum *ibid.* **B 397** (1997) 357;
J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, *Lepton flavor violation in the supersymmetric standard model with seesaw induced neutrino masses*, *Phys. Lett.* **B 357** (1995) 579 [hep-ph/9501407].

[17] A. Bartl, T. Gajdosik, E. Lunghi, A. Masiero, W. Porod, H. Stremnitzer and O. Vives, *General flavor blind MSSM and CP-violation*, *Phys. Rev.* **D 64** (2001) 076009 [hep-ph/0103324].

[18] J. Hisano and D. Nomura, *Solar and atmospheric neutrino oscillations and lepton flavor violation in supersymmetric models with the right-handed neutrinos*, *Phys. Rev.* **D 59** (1999) 116005 [hep-ph/9810479].

[19] An incomplete list of references: S.F. King and I.N.R. Peddie, *Lepton flavour violation from yukawa operators, supergravity and the see-saw mechanism*, *Nucl. Phys.* **B 678** (2004) 339 [hep-ph/0307091];
J.A. Casas and A. Ibarra, *Oscillating neutrinos and $\mu \to e, \gamma$*, *Nucl. Phys.* **B 618** (2001) 171 [hep-ph/0103065];
A. Kageyama, S. Kaneko, N. Shimoyama and M. Tanimoto, *Lepton flavor violating process in degenerate and inverse-hierarchical neutrino models*, *Phys. Lett.* **B 527** (2002) 206 [hep-ph/0110283];

F. Deppisch, H. Pas, A. Redelbach, R. Ruckl and Y. Shimizu, *Probing the Majorana mass scale of right-handed neutrinos in MSUGRA*, *Eur. Phys. J.* **C 28** (2003) 365 [hep-ph/0206122];
S. Lavignac, I. Masina and C.A. Savoy, *$\tau \to \mu\gamma$ and $\mu \to e\gamma$ as probes of neutrino mass models*, *Phys. Lett.* **B 520** (2001) 269 [hep-ph/0106245];
S. Lavignac, I. Masina and C.A. Savoy, *Large solar angle and seesaw mechanism: a bottom-up perspective*, *Nucl. Phys.* **B 633** (2002) 139 [hep-ph/0202086];
J.R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, *A new parametrization of the seesaw mechanism and applications in supersymmetric models*, *Phys. Rev.* **D 66** (2002) 115013 [hep-ph/0206110];
A. Masiero, S. Profumo, S.K. Vempati and C.E. Yaguna, *Lepton flavor violation, neutralino dark matter and the reach of the LHC*, *JHEP* **03** (2004) 046 [hep-ph/0401138];
K. Tobe, J.D. Wells and T. Yanagida, *Neutrino induced lepton flavor violation in gauge-mediated supersymmetry breaking*, *Phys. Rev.* **D 69** (2004) 035010 [hep-ph/0310148];
L.J. Hall and Y. Nomura, *A complete theory of grand unification in five dimensions*, *Phys. Rev.* **D 66** (2002) 075004 [hep-ph/0205067];
H. Abe, K. Choi, K.-S. Jeong and K.-i. Okumura, *Scherk-Schwarz supersymmetry breaking for quasi-localized matter fields and supersymmetry flavor violation*, *JHEP* **09** (2004) 015 [hep-ph/0407005];
S.T. Petcov, S. Profumo, Y. Takanishi and C.E. Yaguna, *Charged lepton flavor violating decays: leading logarithmic approximation versus full RG results*, *Nucl. Phys.* **B 676** (2004) 453 [hep-ph/0306195];
W. Buchmuller, D. Delepine and F. Vissani, *Neutrino mixing and the pattern of supersymmetry breaking*, *Phys. Lett.* **B 459** (1999) 171 [hep-ph/9904219];
W. Buchmuller, D. Delepine and L.T. Handoko, *Neutrino mixing and flavor changing processes*, *Nucl. Phys.* **B 576** (2000) 445 [hep-ph/9912317];
J.R. Ellis and M. Raidal, *Leptogenesis and the violation of lepton number and CP at low energies*, *Nucl. Phys.* **B 643** (2002) 229 [hep-ph/0206174];
S. Pascoli, S.T. Petcov and W. Rodejohann, *On the connection of leptogenesis with low energy CP-violation and LFV charged lepton decays*, *Phys. Rev.* **D 68** (2003) 093007 [hep-ph/0302054];
K.S. Babu, B. Dutta and R.N. Mohapatra, *Lepton flavor violation and the origin of the seesaw mechanism*, *Phys. Rev.* **D 67** (2003) 076006 [hep-ph/0211068];
J. Sato, K. Tobe and T. Yanagida, *A constraint on Yukawa-coupling unification from lepton-flavor violating processes*, *Phys. Lett.* **B 498** (2001) 189 [hep-ph/0010348];
T. Blazek and S.F. King, *Muon anomalous magnetic moment and $\tau \to \mu\gamma$ in a realistic string-inspired model of neutrino masses*, *Phys. Lett.* **B 518** (2001) 109 [hep-ph/0105005];
J.I. Illana and M. Masip, *Quasi-degenerate neutrinos and lepton flavor violation in supersymmetric models*, *Eur. Phys. J.* **C 35** (2004) 365 [hep-ph/0307393];
S.F. King and M. Oliveira, *Lepton flavor violation in string inspired models*, *Phys. Rev.* **D 60** (1999) 035003 [hep-ph/9804283];
J.L. Feng, Y. Nir and Y. Shadmi, *Neutrino parameters, abelian flavor symmetries and charged lepton flavor violation*, *Phys. Rev.* **D 61** (2000) 113005 [hep-ph/9911370];
J.R. Ellis, M.E. Gomez, G.K. Leontaris, S. Lola and D.V. Nanopoulos, *Charged lepton flavour violation in the light of the Super-Kamiokande data*, *Eur. Phys. J.* **C 14** (2000) 319 [hep-ph/9911459];
D.F. Carvalho, J.R. Ellis, M.E. Gomez and S. Lola, *Charged-lepton-flavour violation in the CMSSM in view of the muon anomalous magnetic moment*, *Phys. Lett.* **B 515** (2001) 323

[hep-ph/0103256];
J. Sato and K. Tobe, *Neutrino masses and lepton-flavor violation in supersymmetric models with lopsided Froggatt-Nielsen charges*, *Phys. Rev.* **D 63** (2001) 116010 [hep-ph/0012333];
J.I. Illana and M. Masip, *Lepton flavor violation in Z and lepton decays in supersymmetric models*, *Phys. Rev.* **D 67** (2003) 035004 [hep-ph/0207328];
J. Cao, Z. Xiong and J.M. Yang, *Lepton flavor violating Z decays in supersymmetric see-saw model*, *Eur. Phys. J.* **C 32** (2004) 245 [hep-ph/0307126];
A. Rossi, *Supersymmetric seesaw without singlet neutrinos: neutrino masses and lepton-flavour violation*, *Phys. Rev.* **D 66** (2002) 075003 [hep-ph/0207006];
A. Brignole and A. Rossi, *Flavour non-conservation in goldstino interactions*, *Nucl. Phys.* **B 587** (2000) 3 [hep-ph/0006036].

[20] For a study of LFV at future colliders, see for example: N. Arkani-Hamed, H.-C. Cheng, J.L. Feng and L.J. Hall, *Probing lepton flavor violation at future colliders*, *Phys. Rev. Lett.* **77** (1996) 1937 [hep-ph/9603431];
J. Hisano, M.M. Nojiri, Y. Shimizu and M. Tanaka, *Lepton flavor violation in the left-handed slepton production at future lepton colliders*, *Phys. Rev.* **D 60** (1999) 055008 [hep-ph/9808410];
F. Deppisch, H. Pas, A. Redelbach, R. Ruckl and Y. Shimizu, *The SUSY seesaw model and lepton-flavor violation at a future electron positron linear collider*, *Phys. Rev.* **D 69** (2004) 054014 [hep-ph/0310053];
M. Cannoni, S. Kolb and O. Panella, *Lepton flavour violation in* $e^{\pm}e^{-} \to \ell^{\pm}e^{-}$ $(\ell = \mu, \tau)$ *induced by R-conserving supersymmetry*, *Phys. Rev.* **D 68** (2003) 096002 [hep-ph/0306170];
S.N. Gninenko, M.M. Kirsanov, N.V. Krasnikov and V.A. Matveev, *Probing lepton flavour violation in* $\nu_\mu + N \to \tau + \cdots$ *scattering and* $\mu \to \tau$ *conversion on nucleons*, *Mod. Phys. Lett.* **A 17** (2002) 1407 [hep-ph/0106302];
M. Sher and I. Turan, $\mu + N \to \tau + N$ *at a muon or neutrino factory*, *Phys. Rev.* **D 69** (2004) 017302 [hep-ph/0309183];
S. Kanemura, Y. Kuno, M. Kuze and T. Ota, *A study of lepton flavor violating* $\mu N(eN) \to \tau X$ *reactions in supersymmetric models*, *Phys. Lett.* **B 607** (2005) 165 [hep-ph/0410044].

[21] For a recent review, see A. Masiero and O. Vives, *New physics in CP-violation experiments*, *Ann. Rev. Nucl. Part. Sci.* **51** (2001) 161 [hep-ph/0104027]; *Flavour and CP violation in supersymmetry*, *New J. Phys.* **4** (2002) 4.

[22] For a recent review, please see D.J.H. Chung, L.L. Everett, G.L. Kane, S.F. King, J. Lykken and L.T. Wang, *The soft supersymmetry-breaking lagrangian: theory and applications*, *Phys. Rept.* **407** (2005) 1 [hep-ph/0312378].

[23] R. Barbieri and L.J. Hall, *Signals for supersymmetric unification*, *Phys. Lett.* **B 338** (1994) 212 [hep-ph/9408406];
R. Barbieri, L.J. Hall and A. Strumia, *Violations of lepton flavor and CP in supersymmetric unified theories*, *Nucl. Phys.* **B 445** (1995) 219 [hep-ph/9501334].

[24] A. Masiero, S.K. Vempati and O. Vives, *Seesaw and lepton flavour violation in SUSY so(10)*, *Nucl. Phys.* **B 649** (2003) 189 [hep-ph/0209303]; *SUSY seesaw and FCNC*, *Nucl. Phys.* **137** *(Proc. Suppl.)* (2004) 156 [hep-ph/0405017].

[25] See for example, A. Brignole, L.E. Ibáñez and C. Muñoz, *Soft supersymmetry-breaking terms from supergravity and superstring models*, hep-ph/9707209 and references therein.

[26] E. Dudas, S. Pokorski and C.A. Savoy, *Soft scalar masses in supergravity with horizontal* $\mathrm{U}(1)_X$ *gauge symmetry*, *Phys. Lett.* **B 369** (1996) 255 [hep-ph/9509410].

[27] S.A. Abel and G. Servant, *CP and flavor in effective type-I string models*, *Nucl. Phys.* **B 611** (2001) 43 [hep-ph/0105262];
G.G. Ross and O. Vives, *Yukawa structure, flavour and CP-violation in supergravity*, *Phys. Rev.* **D 67** (2003) 095013 [hep-ph/0211279].

[28] S.F. King, I.N.R. Peddie, G.G. Ross, L. Velasco-Sevilla and O. Vives, *Kaehler corrections and softly broken family symmetries*, *JHEP* **07** (2005) 049 [hep-ph/0407012].

[29] See for example, R. Kitano, M. Koike and Y. Okada, *Detailed calculation of lepton flavor violating muon electron conversion rate for various nuclei*, *Phys. Rev.* **D 66** (2002) 096002 [hep-ph/0203110];
R. Kitano, M. Koike, S. Komine and Y. Okada, *Higgs-mediated muon electron conversion process in supersymmetric seesaw model*, *Phys. Lett.* **B 575** (2003) 300 [hep-ph/0308021] and references therein

[30] K.S. Babu and C. Kolda, *Higgs-mediated* $\tau \to 3\mu$ *in the supersymmetric seesaw model*, *Phys. Rev. Lett.* **89** (2002) 241802 [hep-ph/0206310];
A. Dedes, J.R. Ellis and M. Raidal, *Higgs mediated* $B^0_{s,d} \to \mu\tau, e\tau$ *and* $\tau \to 3\mu, e\mu\mu$*, decays in supersymmetric seesaw models*, *Phys. Lett.* **B 549** (2002) 159 [hep-ph/0209207].

[31] M. Sher, $\tau \to \mu\eta$ *in supersymmetric models*, *Phys. Rev.* **D 66** (2002) 057301 [hep-ph/0207136].

[32] For a comprehensive analysis of $\mu - \tau$ phenomenology, please see: A. Brignole and A. Rossi, *Anatomy and phenomenology of* $\mu\tau$ *lepton flavour violation in the MSSM*, *Nucl. Phys.* **B 701** (2004) 3 [hep-ph/0404211];
A. Brignole and A. Rossi, *Lepton flavour violating decays of supersymmetric Higgs bosons*, *Phys. Lett.* **B 566** (2003) 217 [hep-ph/0304081].

[33] P. Paradisi, *Higgs-mediated* $\tau \to \mu$ *and* $\tau \to e$ *transitions in II Higgs doublet model and supersymmetry*, hep-ph/0508054.

[34] M. Ciuchini, A. Masiero, L. Silvestrini, S.K. Vempati and O. Vives, *Grand unification of quark and lepton fcncs*, *Phys. Rev. Lett.* **92** (2004) 071801 [hep-ph/0307191].

[35] M. Ciuchini, A. Masiero, P. Paradisi, L. Silvestrini, S.K. Vempati and O. Vives, to appear.

[36] See for example, S. Profumo and C.E. Yaguna, *Non-universal gaugino masses and the fate of* $\mu \to e\gamma$, *Nucl. Phys.* **B 681** (2004) 247 [hep-ph/0307225].